## Algebra 1, Quarter 1, Unit 1.1

# Looking at Number Sense

## Overview

**Number of instructional days:** 5 (1 day = 45–60 minutes)

### Content to be learned

- Choose appropriate units consistent with formulas.
- Choose and interpret the scale and origin in graph and data displays.
- Use appropriate measurement when reporting quantities.
- Explain why the sum (or products) of two rational numbers is rational.
- Explain why the sum of a rational and irrational number is irrational.
- Explain why the product of a nonzero rational number and an irrational number is an irrational number.

### Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Use scales and units to make graphs that appropriately represent data.

Reason abstractly and quantitatively.

- Create a coherent representation of problems at hand.
- Understand the quantities being used, not just how to compute them.

Attend to precision.

- Specify units of measure and label axes to clarify the correspondence with quantities in a problem.
- Work collaboratively to communicate content to others with precision.

### Essential questions

- What is the relationship between irrational and rational numbers within the real number system?
- How do you determine the appropriate graphical display for a given problem?
- Why are units important in the problem-solving process?
- How do you determine the appropriate level of accuracy for a quantitative solution?
- What type of real numbers would a student find in applying the arithmetic operations of addition and multiplication to any combination of rational and irrational numbers?

# Written Curriculum

**Common Core State Standards for Mathematical Content**

| **Quantities★** | **N-Q** |
|---|---|

**Reason quantitatively and use units to solve problems.** [*Foundation for work with expressions, equations, and functions*]

N-Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.★

N-Q.2 Define appropriate quantities for the purpose of descriptive modeling.★

N-Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.★

| **The Real Number System** | **N-RN** |
|---|---|

**Use properties of rational and irrational numbers.**

N-RN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

**Common Core Standards for Mathematical Practice**

## 1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

## 2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

## 6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

## Clarifying the Standards

*Prior Learning*

In kindergarten, students described and compared measurable attributes. In grade 1, students measured lengths and indirectly and by iterating length units. They also represented and interpreted data. In grade 2, students measured and estimated lengths in standard units. They also related addition and subtraction to length.

In grade 3, students began developing an understanding of rational numbers (fractions). In grade 4, students extended their understanding of rational numbers. Also in grade 4, students solved problems involving measurement and measurement conversion.

In grade 5, rational numbers were extended to using equivalent fractions for adding and subtracting. Students were also responsible for becoming fluent in multiplying and dividing rational numbers. They analyzed the relationship between independent and dependent variables; and converted like measurement units within a given measurement.

In grade 6, students were introduced to systems of rational numbers. They were responsible for becoming fluent in multiplying and dividing rational numbers. They analyzed the relationship between independent and dependent variables.

In grade 7, students applied and expanded previous understanding of rational numbers (in real-world settings and/or with equations). In grade 8, students were introduced to irrational numbers. They converted rational numbers to show their decimal form and approximated irrational numbers.

*Current Learning*

Algebra I students extend their knowledge of rational and irrational numbers to explain why the sum or product of two rational or irrational numbers is rational or irrational. Students learn how to interpret units, choose appropriate scales, and create the coordinate system for graphs and/or data displays. These skills will be revisited in more detail throughout the course. Students also define the meaning of variables in the context of given problems. They are exposed to determining appropriate limitations on measurements when reporting quantities (knowing when to round up or down within the context of a given problem).

*Future Learning*

Algebra 2 students will perform arithmetic operations involving complex numbers. They will also use complex numbers in polynomial identities and equations. Students in fourth-year courses will represent and model quantities with vectors and matrices.

Rational and irrational numbers will be used in all future courses. Career readiness connections include professions such as accountants, bankers, construction workers, oilfield workers, mechanics, tax preparers, and small business owners.

## Additional Findings

None listed.

## Algebra 1, Quarter 1, Unit 1.2

# Interpreting Algebraic Expressions

## Overview

**Number of instructional days:** 7 (1 day = 45–60 minutes)

### Content to be learned

- Interpret parts of a linear expression.
- Interpret complicated linear expressions by viewing one or more parts as a single entity.
- Use the structure of a linear expression to change its format through factoring.
- Understand linear expressions are closed under certain operations.

### Mathematical practices to be integrated

Model with mathematics.

- Solve real-world math problems using linear expressions.
- Use algebra tiles to perform basic arithmetic on linear expressions.

Use appropriate tools strategically.

- Use paper, pencil, and algebra tiles to perform basic arithmetic on linear expressions.

Attend to precision.

- Students accurately define parts of linear expressions.
- Students use think-pair-share to simplify linear expressions.

### Essential questions

- Under what arithmetic operations are linear expressions closed?
- Which arithmetic properties can be used to evaluate expressions?
- Why are degree, coefficients, terms, factors, and variables, important in the study of polynomials?
- How do you show that two algebraic expressions are equivalent?

# Written Curriculum

## Common Core State Standards for Mathematical Content

| **Seeing Structure in Expressions** | **A-SSE** |
|---|---|

**Interpret the structure of expressions** [*Linear,* ~~*exponential, quadratic*~~]

A-SSE.1 Interpret expressions that represent a quantity in terms of its context.★

a. Interpret parts of an expression, such as terms, factors, and coefficients.

b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P.*

A-SSE.2 Use the structure of an expression to identify ways to rewrite it. ~~*For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$~~*. (Linear Only)*

| **Arithmetic with Polynomials and Rational Expressions** | **A-APR** |
|---|---|

**Perform arithmetic operations on polynomials** [*Linear* ~~*and quadratic*~~]

A-APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

## Common Core Standards for Mathematical Practice

### 4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

## 5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

## 6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

## Clarifying the Standards

*Prior Learning*

In kindergarten, students used objects to represent adding and subtracting as expressions. In grade 5, students wrote and interpreted numerical expressions In grade 6, students identified parts of an expression containing whole numbers. They applied the properties of operations to generate equivalent expressions. Students also identified when two expressions are equivalent (for example, $y + y+ y = 3y$).

In grade 7, students applied properties of operations as strategies to add, subtract, factor, and expand linear expressions. They also understood that rewriting an expression in different forms can show how quantities are related. In grade 8, students applied the properties of integer exponents to generate equivalent numerical expressions.

*Current Learning*

Performing arithmetic operations on polynomials is a fluency standard for algebra I. Students become fluent in adding, subtracting, and multiplying polynomials, which will support their work throughout the study of algebra.

The ability to identify structure in expressions by viewing one or more parts as single entities is also a fluency standard in algebra 1. Students become fluent in transforming expressions and chunking, which are essential to factoring later in algebra 1. Students interpret the parts of simple and more complicated expressions from a given context, which must be modeled by students. They simplify and rewrite expressions by combining like terms, using properties as a means to accomplish this objective. Students also learn and understand that polynomials are closed within the operations of addition, subtraction, and multiplication.

*Future Learning*

In algebra 2, students will be interpreting the structure of polynomial and rational expressions. Also, they will take their ability to perform arithmetic operations on polynomials and extend it beyond quadratics. In addition, students will learn how to divide polynomials to determine the zeros of the function. Students will use polynomial identities to describe numerical relationships. A career readiness connection can be made to business owners, bankers, accountants, engineers, and architects.

## Additional Findings

Students have difficulty thinking of algebraic expressions and variables to represent a range of values. *Adding It Up* describes a study designed to address these difficulties, in which “students were asked to give instructions to an ‘idealized mathematics machine.” The students easily made sense of the idea of employing letters to write rules that would enable the machine to solve whole classes of problems.” (p. 264)

### Algebra 1, Quarter 1, Unit 1.3

# Operations with Linear Functions and Inequalities

## Overview

**Number of instructional days:** 15 (1 day = 45–60 minutes)

### Content to be learned

- Understand functions and function notation.
- Evaluate functions for inputs in their domains.
- Interpret intercepts and where functions increase or decrease, and are positive or negative.
- Determine an applicable domain for a function.
- Calculate and interpret the average rate of change of a function.
- Graph simple functions by hand and complicated functions using technology.
- Compare properties of two functions.
- Introduce explicit expressions and the recursive process.
- Construct linear and exponential functions.
- Observe using graphs and tables that a quantity increases exponentially.
- Interpret the parameters in a linear or exponential function.

### Mathematical practices to be integrated

Model with mathematics.

- Graph functions by hand and using technology.

Use appropriate tools strategically.

- Use pencil, paper, graphing calculators, and graph paper to graph functions.

Attend to precision.

- Assess functions for inputs in their domains.

Look for and make use of structure.

- Prove that linear functions grow by equal differences over equal intervals.

### Essential questions

- What are the connections between the multiple representations of linear functions?
- How does a function model a relationship between two quantities?
- What is the difference between a function and an equation?
- What type of dependency exists between the domain and the range of linear and exponential functions?
- Why is function notation important?

# Written Curriculum

## Common Core State Standards for Mathematical Content

**Interpreting Functions** **F-IF**

**Understand the concept of a function and use function notation** [*Learn as general principle; focus on linear ~~and exponential and on arithmetic and geometric~~ sequences*]

F-IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$. The graph of $f$ is the graph of the equation $y = f(x)$.

F-IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F-IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$.

**Interpret functions that arise in applications in terms of the context** [*Linear, ~~exponential, and quadratic~~*]

F-IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; ~~intervals~~ where the function is increasing, decreasing, positive, or negative; ~~relative maximums and minimums; symmetries; end behavior; and periodicity.*~~*

F-IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★

F-IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

**Analyze functions using different representations** [*Linear, ~~exponential, quadratic, absolute value, step, piecewise-defined~~*]

F-IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

a. Graph linear ~~and quadratic~~ functions and show intercepts, ~~maxima, and minima.~~

F-IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

| **Building Functions** | **F-BF** |
|---|---|

**Build a function that models a relationship between two quantities** [*For F.BF.1, 2, linear, ~~exponential, and quadratic~~*]

F-BF.1 Write a function that describes a relationship between two quantities.★

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

b. Combine standard function types using arithmetic operations. ~~*For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*~~

**Build new functions from existing functions** [*Linear, ~~exponential, quadratic, and absolute value; for F.BF.4a, linear only~~*]

F-BF.3 Identify the effect on the graph of replacing *f(x)* by *f(x) + k, k f(x), f(kx),* and *f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. ~~*Include recognizing even and odd functions from their graphs and algebraic expressions for them.*~~

| **Linear, Quadratic, and Exponential Models★** | **F-LE** |
|---|---|

**Construct and compare linear, ~~quadratic, and exponential~~ models and solve problems**

F-LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.★

a. Prove that linear functions grow by equal differences over equal intervals~~, and that exponential functions grow by equal factors over equal intervals~~.

b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

F-LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).★

F-LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.★

**Interpret expressions for functions in terms of the situation they model**

F-LE.5 Interpret the parameters in a linear or exponential function in terms of a context.★

**Common Core Standards for Mathematical Practice**

## 4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

## 5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

## 6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

## 7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see $7 \times 8$ equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as $2 \times 7$ and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers $x$ and $y$.

## Clarifying the Standards

*Prior Learning*

In kindergarten, students began describing and comparing data. In grades 1–5, students represented and interpreted data. Second grade students graphed picture and bar graphs. In third grade, students solved one- and two-step problems using information presented in a scaled bar graph. Fifth grade students graphed ordered pairs on a coordinate plane. In sixth grade, they represented and analyzed quantitative relationships between dependent and independent variables.

Students continued graphing linear equations in grade 7. In eighth grade, students analyzed and solved linear equations and pairs of simultaneous linear equations. They used functions to model relationships between quantities. Students also learned that a function is a rule that assigns each input to exactly one output. They learned how to graph a function using a set of ordered pairs, and they learned that functions can be presented in different ways, including graphically, numerically in tables, or by verbal description. Students also learned how to interpret a slope–intercept form equation ($y = mx + b$) and that this type of equation represents a linear function. They also learned to determine when a function is not linear.

*Current Learning*

In algebra 1, students understand the concept of functions and function notation. They evaluate and graph functions by hand and using graphing calculators. Students interpret intercepts (positive and negative slopes). Students use appropriate data display and they compare properties of two functions. Also, students learn function notation, $f(x)$.

Students learn that functions represent real-world situations with specific restrictions. They learn how to evaluate, interpret, and make predictions from a given context. They also learn how to determine from a table whether a function exists, and then identify the domains and ranges of the function from the table. Students learn how to develop an explicit or recursive formula, given a sequence of integers. They learn the differences between explicit and recursive sequences. In algebra I, students must compare and contrast relationships between two functions; these skills will be used throughout algebra I.

*Future Learning*

In algebra 2, students interpret functions that arise in applications in terms of a context. They also analyze functions using different representations in algebra 2. In fourth courses, students will use logarithmic and trigonometric functions. Career readiness is relevant for computer programmers, farmers, health information technicians, actuaries, and industrial, nuclear, and petroleum engineers.

## Additional Findings

Students have difficulty understanding what the solution means in the context of the problem. This can be clarified by connecting different representations of the problem. "Students translate among verbal, tabular, graphical, and algebraic representations of functions, and they describe how such aspects of a function as slope and *y*-intercept appear in different representations." (*Curriculum Focal Points*, p. 20)

### Algebra 1, Quarter 1, Unit 1.4

# Solving Linear Equations and Inequalities

## Overview

**Number of instructional days:** 6 (1 day = 45–60 minutes)

### Content to be learned

- Justify each step in solving a simple equation.
- Prove solution methods using a viable argument.
- Solve linear equations and inequalities in one variable, including literal equations.
- Understand that the graph of a linear equation represents the relationship of two variables.
- Use word problems to create equations and inequalities.
- Create and graph on a coordinate plane equations with independent and dependent variables.

### Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Explain the steps they took in order to solve a linear equation or inequality.
- Prove that the steps they took to solve a problem are viable through various methods.

Model with mathematics.

- Solve problems arising in everyday life, society, and the workplace by creating equations and inequalities.
- Create and interpret information on a graph of equations and inequalities.
- Use appropriate tools strategically.
- Use pencil and paper, concrete models, a ruler, a calculator, or a computer algebra system to graph equations on coordinate axes with labels and scales.

### Essential questions

- What are the similarities and differences between solving a problem algebraically and solving it graphically?
- How does rearranging formulas help you solve real-world problems?
- How does justifying your steps help to determine that your solution to the equation is correct?
- What are the similarities and differences in solving one-variable and two-variable equations?

# Written Curriculum

**Common Core State Standards for Mathematical Content**

| **Reasoning with Equations and Inequalities** | **A-REI** |
|---|---|

**Understand solving equations as a process of reasoning and explain the reasoning**

A-REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

**Solve equations and inequalities in one variable**

A-REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

**Represent and solve equations and inequalities graphically** [*Linear and ~~exponential~~; learn as general principle*]

A-REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

| **Creating Equations★** | **A-CED** |
|---|---|

**Create equations that describe numbers or relationships** [*Equations using all available types of expressions, including simple root functions*]

A-CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear ~~and quadratic~~ functions~~, and simple rational and exponential~~ functions.*★

A-CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

A-CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*★

**Common Core Standards for Mathematical Practice**

## 1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

### 4 Model with mathematics.

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### 5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

## Clarifying the Standards

*Prior Learning*

In kindergarten, students explained thinking by making a drawing. In first grade, students began to solve simple word problems. In third grade, students solved two-step word problems using letters for unknown quantities. In fourth grade, students began solving multistep word problems. In fifth grade, students graphed the ordered pairs on a coordinate plane. In grade 6, students learned that the solutions to an equation are the values of the variables that make the equation true. Students used properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. They also constructed and analyzed tables, such as tables of quantities that were in equivalent ratios, and they used equations (such as $3x = y$) to describe relationships between quantities. In grade 7, students used the arithmetic of rational numbers as they formulated expressions and equations in one variable and used these equations to solve problems. Seventh-grade students graphed proportional relationships and understood the unit rate informally as a measure of the steepness of the related line, called the slope. In grade 8, students used linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognized equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality ($m$) is the slope, and the graphs are lines through the origin. They understood that the slope ($m$) of a line is a constant rate of change, so that, if the input, or $x$-coordinate, changes by an amount $A$, the output, or $y$-coordinate, changes by the amount $(m)A$. Interpreting the model in the context of the data required students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and $y$-intercept) in terms of the situation. Eighth-grade students also strategically chose and efficiently implemented procedures to solve linear equations in one variable, understanding that when they used the properties of equality and the concept of logical equivalence, they maintained the solutions of the original equation. Students used linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

*Current Learning*

The content and concepts in this unit are a critical area for algebra 1 students. By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. This unit builds on these earlier experiences by asking students to analyze and explain the process of solving an equation. Students develop fluency in writing, interpreting, and translating among various forms of linear equations and using them to solve problems. They master the solution of linear equations and apply related solution techniques. All of this work is grounded in understanding quantities and relationships between them. Students should focus on and master standard A.REI.1 for linear equations and be able to extend and apply their reasoning to other types of equations in future courses.

*Future Learning*

In geometry, students will use equations to help solve problems with congruence and proportions. Students will solve equations and the process of reasoning will be expanded in algebra 2 for extraneous solutions. Students in algebra 2 will expand their ability to create equations that describe numbers or relationships to include root functions. They will also build new functions from existing functions, including simple radical, rational, and exponential functions, with an emphasis on the common effect of each transformation across function types. Students will solve exponential equations with logarithms in algebra 2. Solving systems of equations is used in all subsequent mathematical courses. Engineers in every field, computer programmers and software engineers, biological scientists, economists, urban and regional planners, and optometrists and pharmacists use these methods in their careers.

## Additional Findings

In *A Research Companion to Principles and Standards for School Mathematics,* Chazan and Yerushalmy discuss the cognitive difficulties that many students have in working with the complex relationships embedded in systems of equations. As an example, they describe the methods that students must use to solve a system of equations consisting of a linear equation in standard form and a circle in standard form. As students work through solving such a system, they must move “from an equation in two variables to a function of one” to enable use of the substitution algorithm “from an equation in two variables to an equation in one variable” *using* the algorithm, to generating equivalent expressions in solving the new equation. Chazan and Yerushalmy indicate that this complexity is common in learning about equivalence in school algebra, and that this cognitive complexity must be taken into account when approaching topics involving equivalence (pp.129-131). Chazan and Yerushalmy also indicate that graphing technology can assist students in making sense of equivalent expressions. (p. 130)

### Algebra 1, Quarter 1, Unit 1.5

# Modeling Linear Equations and Inequalities

## Overview

**Number of instructional days:** 6 (1 day = 45–60 minutes)

### Content to be learned

- Represent equations and inequalities through modeling.
- Determine if solutions are viable or nonviable.
- Solve systems of equations using substitution only.
- Graph linear inequalities.
- Graph a system of linear inequalities.

### Mathematical practices to be integrated

Model with mathematics.

- Graph linear equations and inequalities.

Use appropriate tools strategically.

- Students will use pencil, paper or graphing paper, graphing calculators, and ruler to graph equations and inequalities.

### Essential questions

- How do you know when your solution is reasonable in the context of a given problem?
- What are the similarities and differences in the ways to solve linear systems?
- How is solving an inequality similar to and different from solving an equation?
- How do you use inequalities to represent constraints in real-world problems?

# Written Curriculum

## Common Core State Standards for Mathematical Content

| **Creating Equations★** | **A-CED** |
|---|---|

**Create equations that describe numbers or relationships** [*Equations using all available types of expressions, including simple root functions*]

A-CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods*.★

| **Reasoning with Equations and Inequalities** | **A-REI** |
|---|---|

**Solve systems of equations** [*Linear-linear ~~and linear-quadratic~~*]

A-REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line* $y = -3x$ and the circle $x^2 + y^2 = 3$.

A-REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

## Common Core Standards for Mathematical Practice

### 4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

## 5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

## Clarifying the Standards

*Prior Learning*

In third grade, students solved simple equations and assessed reasonableness of answers. In grades 4–5, students solved word problems involving equations. In grade 5, students graphed ordered pairs on a coordinate plane. In grade 6, students solved real-world problems by graphing points in all four quadrants of a coordinate plane. They also mastered reasoning about and solving one-variable equations and inequalities. In seventh grade, students mastered solving real-life and mathematical problems using numerical and algebraic expressions and equations. In eighth grade, students mastered analyzing and solving linear equations and pairs of simultaneous linear equations. They made connections among proportional relationships, lines, and linear equations, but did not work directly with inequalities.

*Current Learning*

In algebra 1, students create and solve linear inequalities with one variable and use them to solve problems. They interpret solutions as viable or nonviable. Students graph solutions to linear inequalities in two variables as a half plane. Students represent constraints by equations and inequalities and by systems of equations and inequalities. Students solve simple systems of equations graphically and algebraically using substitution only. Students graph linear inequalities in two variables as a half-plane.

*Future Learning*

In algebra 2, students will use all types of functions to create inequalities including root functions. Inequalities will be used in all subsequent mathematical courses. As career readiness, students could use these concepts as a budget analyst, engineer, economist, medical records technician, health information technician, or medical assistant.

## Additional Findings

In *A Research Companion to Principles and Standards for School Mathematics,* Chazan and Yerushalmy discuss the cognitive difficulties that many students have in working with the complex relationships embedded in systems of equations. As an example, they describe the methods that students must use to solve a system of equations consisting of a linear equation in standard form and a circle in standard form. As students work through the solving of such a system, they must move "from an equation in two variables to a function of one" to enable use of the substitution algorithm "from an equation in two variables to an equation in one variable" *using* the algorithm, to generating equivalent expressions in solving the new equation. Chazan and Yerushalmy indicate that this complexity is common in learning about equivalence in school algebra, and that this cognitive complexity must be taken into account when approaching topics involving equivalence (pp.129–131). Chazan and Yerushalmy also indicate that graphing technology can assist students in making sense of equivalent expressions. (p. 130)